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Study of the η-η' system in the two mixing angle scheme

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Abstract: An analysis of various decay processes is performed using the two mixing angle description of the η-η' system, incorporating the link to the gluonic sector through anomalies. The agreement is excellent. For comparison with previous works, our results are expressed both in the "octet-singlet" and in the "quark-flavour" basis. It turns out that at the present experimental accuracy, the two angles are significantly different in the former, but not in the latter basis. The implications of our analysis for the large N_c χPT predictions are also discussed.

Keywords: Electromagnetic Processes and Properties, Phenomenological Models.

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Contents

1. Introduction

The phenomenon of mixing in the η-η' system has attracted much attention since the advent of SU$_f(3)$ symmetry. Conventionally, one regards η and η' as linear combinations of octet and singlet basis states, parametrized by a mixing angle θ_P. A determination of its value can be achieved either diagonalizing a mass matrix, as done in Chiral Perturbation Theory (χPT), or from phenomenology [1]. Several exhaustive analyses surveying many different processes have been performed along the years with values for the mixing angle ranging from $-24°$ to $-10°$ [2]–[10]. In all these previous analyses the dependence with energy of the η-η' mixing angle has been neglected. A recent analysis assuming such an energy dependence is also present in the literature [11].

The η-η' mixing is strongly connected with the U(1) anomaly of QCD. Recently, it has pointed out that an extension of χPT taking into account the effects of the chiral anomaly through a perturbative expansion leads to a description of the η-η' system in terms of two mixing angles [12]. In this framework, the four pseudoscalar decay constants associated to the matrix elements of axial-vector currents are written in terms of two basic decay constants f_8, f_0 and two angles θ_8, θ_0 [13]. Alternative approaches also including the chiral anomaly *ab initio* but using a more conventional one mixing angle scheme are discussed in refs. [4, 14−17]. On the phenomenological side, the idea of using a two mixing angle scheme has been explored in refs. [18, 19].

The aim of this work is to perform an updated phenomenological analysis of various decay processes using the two mixing angle description of the η-η' system. The analysis is carried out in the octet-singlet and quark-flavour basis and our theoretical predictions compared with the latest experimental data. The comparison will serve us to check the validity of the two mixing angle scheme and its improvement over the standard one angle picture. The analysis also tests the sensitivity to the mixing angle schemes.

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In section 2, we shortly introduce the notation used in the analysis. In section 3, we compute the radiative decays $(\eta,\eta')\to\gamma\gamma$ and the ratio $R_{J/\psi}\equiv\Gamma(J/\psi\to\eta'\gamma/\eta\gamma)$ in the two mixing angle scheme of both the octet-singlet and quark-flavour basis in order to obtain in each basis the preferred values for the mixing angles involved in the analysis. Section 4 is devoted to the consequences of our approach in the context of the radiative decays of the lowest-lying vector and pseudoscalar mesons, $V\to P\gamma$ and $P\to V\gamma$, respectively. In section 5, we compare our best results for the mixing parameters with the theoretical expectations of large N_c χPT and previous phenomenological analyses. The implications of our analysis on these large N_c χPT predictions are also discussed. Finally, in section 6, we present our conclusions.

2. Notation

The decay constants of the η-η' system in the octet-singlet basis f_P^a $(a=8,0;P=\eta,\eta')$ are defined as[1]

$$\langle 0|A_\mu^a|P(p)\rangle = i f_P^a p_\mu\,, \tag{2.1}$$

where $A_\mu^{8,0}$ are the octet and singlet axial-vector currents whose divergences are

$$\begin{aligned}\partial^\mu A_\mu^8 &= \frac{2}{\sqrt{6}}(m_u\bar{u}i\gamma_5 u + m_d\bar{d}i\gamma_5 d - 2m_s\bar{s}i\gamma_5 s)\,,\\ \partial^\mu A_\mu^0 &= \frac{2}{\sqrt{3}}(m_u\bar{u}i\gamma_5 u + m_d\bar{d}i\gamma_5 d + m_s\bar{s}i\gamma_5 s) + \frac{1}{\sqrt{3}}\frac{3\alpha_s}{4\pi}G_{\mu\nu}^a\tilde{G}^{a,\mu\nu}\,,\end{aligned} \tag{2.2}$$

where $G_{\mu\nu}^a$ is the gluonic field-strength tensor and $\tilde{G}^{a,\mu\nu}\equiv\frac{1}{2}\epsilon^{\mu\nu\alpha\beta}G_{\alpha\beta}^a$ its dual. The divergence of the matrix elements (2.1) are then written as

$$\langle 0|\partial^\mu A_\mu^a|P\rangle = f_P^a m_P^2\,, \tag{2.3}$$

where m_P is the mass of the pseudoscalar meson.

Each of the two mesons $P=\eta,\eta'$ has both octet and singlet components, $a=8,0$. Consequently, eq. (2.1) defines *four independent* decay constants. Following the convention of refs. [12, 13] the decay constants are parameterized in terms of two basic decay constants f_8, f_0 and two angles θ_8,θ_0

$$\begin{pmatrix} f_\eta^8 & f_\eta^0 \\ f_{\eta'}^8 & f_{\eta'}^0 \end{pmatrix} = \begin{pmatrix} f_8\cos\theta_8 & -f_0\sin\theta_0 \\ f_8\sin\theta_8 & f_0\cos\theta_0 \end{pmatrix}. \tag{2.4}$$

This parametrization is the most general involving two independent axial-vector currents and two different physical states.

Neglecting the contribution of the *up* and *down* quark masses, as in ref. [14], the matrix elements of the chiral anomaly between the vacuum and (η,η') states are

$$\begin{aligned}\langle 0|\frac{3\alpha_s}{4\pi}G\tilde{G}|\eta\rangle &= \sqrt{\frac{3}{2}}m_\eta^2(f_8\cos\theta_8 - \sqrt{2}f_0\sin\theta_0)\,,\\ \langle 0|\frac{3\alpha_s}{4\pi}G\tilde{G}|\eta'\rangle &= \sqrt{\frac{3}{2}}m_{\eta'}^2(f_8\sin\theta_8 + \sqrt{2}f_0\cos\theta_0)\,.\end{aligned} \tag{2.5}$$

[1] The axial-vector currents are defined as $A_\mu^a = \bar{q}\gamma_\mu\gamma_5\frac{\lambda^a}{\sqrt{2}}q$ with the normalization convention $f_\pi = \sqrt{2}F_\pi = 130.7\,\mathrm{MeV}$.

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Analogously, in the quark-flavour basis the decay constants are parameterized in terms of f_q, f_s and ϕ_q, ϕ_s:

$$\begin{pmatrix} f_\eta^q & f_\eta^s \\ f_{\eta'}^q & f_{\eta'}^s \end{pmatrix} = \begin{pmatrix} f_q \cos\phi_q & -f_s \sin\phi_s \\ f_q \sin\phi_q & f_s \cos\phi_s \end{pmatrix}, \tag{2.6}$$

and the non-strange and strange axial-vector currents are defined as

$$\begin{aligned} A_\mu^q &= \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d) = \frac{1}{\sqrt{3}}(A_\mu^8 + \sqrt{2}A_\mu^0)\,, \\ A_\mu^s &= \bar{s}\gamma_\mu\gamma_5 s = \frac{1}{\sqrt{3}}(A_\mu^0 - \sqrt{2}A_\mu^8)\,. \end{aligned} \tag{2.7}$$

The divergences of these currents are

$$\begin{aligned} \partial^\mu A_\mu^q &= \sqrt{2}(m_u \bar{u} i\gamma_5 u + m_d \bar{d} i\gamma_5 d) + \frac{\sqrt{2}}{3}\frac{3\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}^{a,\mu\nu}\,, \\ \partial^\mu A_\mu^s &= 2m_s \bar{s} i\gamma_5 s + \frac{1}{3}\frac{3\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}^{a,\mu\nu}\,, \end{aligned} \tag{2.8}$$

and therefore the matrix elements of the chiral anomaly in this basis are

$$\langle 0|\frac{3\alpha_s}{4\pi} G\tilde{G}|\eta\rangle = \frac{3}{\sqrt{2}} m_\eta^2 f_q \cos\phi_q\,, \qquad \langle 0|\frac{3\alpha_s}{4\pi} G\tilde{G}|\eta'\rangle = \frac{3}{\sqrt{2}} m_{\eta'}^2 f_q \sin\phi_q\,. \tag{2.9}$$

3. Experimental values for the $\theta_{8,0}$ and $\phi_{q,s}$ mixing angles

In order to reach some predictions from our two mixing angle analysis we must first know the values of θ_8 and θ_0 preferred by the experimental data. We will use as constraints[2] the experimental decay widths of $(\eta, \eta') \to \gamma\gamma$ [1]

$$\begin{aligned} \Gamma(\eta \to \gamma\gamma) &= (0.510 \pm 0.026)\,\mathrm{keV}\,, \\ \Gamma(\eta' \to \gamma\gamma) &= (4.29 \pm 0.15)\,\mathrm{keV}\,. \end{aligned} \tag{3.1}$$

Analogously to the $\pi^0 \to \gamma\gamma$ case, one assumes that the interpolating fields η and η' can be related with the axial-vector currents (see e.g. refs. [14, 15]) in the following way:

$$\begin{aligned} \eta(x) &= \frac{1}{m_\eta^2}\frac{f_{\eta'}^0 \partial^\mu A_\mu^8(x) - f_{\eta'}^8 \partial^\mu A_\mu^0(x)}{f_{\eta'}^0 f_\eta^8 - f_{\eta'}^8 f_\eta^0}\,, \\ \eta'(x) &= \frac{1}{m_{\eta'}^2}\frac{f_\eta^0 \partial^\mu A_\mu^8(x) - f_\eta^8 \partial^\mu A_\mu^0(x)}{f_\eta^0 f_{\eta'}^8 - f_\eta^8 f_{\eta'}^0}\,. \end{aligned} \tag{3.2}$$

This leads to[3]

$$\begin{aligned} \Gamma(\eta \to \gamma\gamma) &= \frac{\alpha^2 m_\eta^3}{96\pi^3}\left(\frac{f_{\eta'}^0 - 2\sqrt{2}f_{\eta'}^8}{f_{\eta'}^0 f_\eta^8 - f_{\eta'}^8 f_\eta^0}\right)^2 = \frac{\alpha^2 m_\eta^3}{96\pi^3}\left(\frac{c\theta_0/f_8 - 2\sqrt{2}s\theta_8/f_0}{c\theta_0 c\theta_8 + s\theta_8 s\theta_0}\right)^2, \\ \Gamma(\eta' \to \gamma\gamma) &= \frac{\alpha^2 m_{\eta'}^3}{96\pi^3}\left(\frac{f_\eta^0 - 2\sqrt{2}f_\eta^8}{f_\eta^0 f_{\eta'}^8 - f_\eta^8 f_{\eta'}^0}\right)^2 = \frac{\alpha^2 m_{\eta'}^3}{96\pi^3}\left(\frac{s\theta_0/f_8 + 2\sqrt{2}c\theta_8/f_0}{c\theta_0 c\theta_8 + s\theta_8 s\theta_0}\right)^2. \end{aligned} \tag{3.3}$$

[2]We choose such constrains because those decays are well understood in terms of the electromagnetic anomaly (see e.g. ref. [20]).

[3]Note that if one assumes a one mixing angle scheme ($\theta_8 = \theta_0 \equiv \theta$) the standard result is obtained (see e.g. ref. [20]).

Because of the four unknown parameters (θ_8, θ_0, f_8 and f_0) that appear in eq. (3.3), in order to get their allowed values we need two additional constraints (apart from the experimental constraints in eq. (3.1)). On the one hand, we will use the well established prediction of χPT: $f_8 = 1.28 f_\pi$ ($f_\pi = 130.7\,\mathrm{MeV}$) as a theoretical constrain (see variants on this later). On the other hand, we will use the experimental value of the ratio [1]

$$R_{J/\psi} \equiv \frac{\Gamma(J/\psi \to \eta'\gamma)}{\Gamma(J/\psi \to \eta\gamma)} = 5.0 \pm 0.6\,. \tag{3.4}$$

According to ref. [21], the radiative $J/\psi \to P\gamma$ decays are dominated by non-perturbative gluonic matrix elements (see ref. [4] for further comments on the accuracy of this statement):

$$R_{J/\psi} = \left| \frac{\langle 0|G\tilde{G}|\eta'\rangle}{\langle 0|G\tilde{G}|\eta\rangle} \right|^2 \left(\frac{p_{\eta'}}{p_\eta} \right)^3 , \tag{3.5}$$

where $p_P = M_{J/\psi}(1 - m_P^2/M_{J/\psi}^2)/2$ is the three-momentum of the P-meson in the rest frame of the decaying J/ψ (with mass $M_{J/\psi}$). Using eq. (2.5) one gets

$$\begin{aligned} R_{J/\psi} &= \left| \frac{m_{\eta'}^2 (f_{\eta'}^8 + \sqrt{2} f_{\eta'}^0)}{m_\eta^2 (f_\eta^8 + \sqrt{2} f_\eta^0)} \right|^2 \left(\frac{p_{\eta'}}{p_\eta} \right)^3 \\ &= \left[\frac{m_{\eta'}^2 (f_8 \sin\theta_8 + \sqrt{2} f_0 \cos\theta_0)}{m_\eta^2 (f_8 \cos\theta_8 - \sqrt{2} f_0 \sin\theta_0)} \right]^2 \left(\frac{p_{\eta'}}{p_\eta} \right)^3 . \end{aligned} \tag{3.6}$$

Comparing the experimental values of $\Gamma(\eta, \eta' \to \gamma\gamma)$ and $R_{J/\psi}$ with the theoretical predictions shown in eqs. (3.3) and (3.6), one obtains

$$\theta_8 = (-22.2 \pm 1.8)^\circ\,, \qquad \theta_0 = (-8.7 \pm 2.1)^\circ\,, \qquad f_0 = (1.18 \pm 0.04) f_\pi\,. \tag{3.7}$$

Using instead of $f_8 = 1.28 f_\pi$ the prediction of large N_c χPT [22, 23]: $f_8 = 1.34 f_\pi$ [13], we would have found

$$\theta_8 = (-22.9 \pm 1.8)^\circ\,, \qquad \theta_0 = (-6.9 \pm 2.0)^\circ\,, \qquad f_0 = (1.20 \pm 0.04) f_\pi\,. \tag{3.8}$$

The previous results are in agreement with the expectations of large N_c χPT [12, 13] and with phenomenological analyses [18, 24, 25] (see also ref. [26] for a comparison of various analyses). Notice that the numerical value of f_0 cannot in principle be determined unambiguously, because it depends on the renormalization scale [27, 28]. However, the scale dependence of the anomalous dimension of the singlet axial-vector current is very weak [12]. Therefore, in our numerical analysis we neglect that dependence and consider f_0 as a constant. The results of eqs. (3.7) and (3.8) constitute the first result of the present analysis. It is worth noting that the θ_8 and θ_0 mixing angle values are different at the 3σ level, while the values of the pseudoscalar decay constant f_0 remain compatible with other results.

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In order to check the consistency of these results one can perform the same analysis but fixing $\theta_8 = \theta_0 \equiv \theta$, i.e. a one mixing angle scheme. Keeping $f_8 = 1.28 f_\pi$ and using $\eta, \eta' \to \gamma\gamma$ in order to fit f_0 and θ, one gets: $\theta = (-22.2 \pm 1.7)^\circ$ and $f_0 = (1.07 \pm 0.04) f_\pi$ — or $\theta = (-22.9 \pm 1.7)^\circ$ and $f_0 = (1.06 \pm 0.04) f_\pi$ if $f_8 = 1.34 f_\pi$. In these cases, however, the predicted value for $R_{J/\psi} \simeq 2.1$ (or $\simeq 1.7$) is incompatible with the experimental result in eq. (3.4). Alternatively, leaving f_8 as a free parameter one gets: $\theta = (-17.7 \pm 1.2)^\circ$, $f_8 = (1.02 \pm 0.06) f_\pi$ and $f_0 = (1.09 \pm 0.04) f_\pi$. In this other case, $R_{J/\psi} \simeq 3.5$ and the value of f_8 is incompatible at the 3σ level with the predictions of χPT and large N_c χPT.

Therefore, a two mixing angle scheme leads to significantly different mixing angles in the octet-singlet basis to describe the radiative $\eta, \eta' \to \gamma\gamma$ decays and the ratio $R_{J/\psi}$ simultaneously.

In the quark-flavour basis (see the appendix for the corresponding theoretical expressions), a simultaneous fit of $\eta, \eta' \to \gamma\gamma$ and $R_{J/\psi}$ with $f_q = f_\pi$ suggests that, at the current experimental precision, $\phi_q \simeq 39.8^\circ$ and $\phi_s \simeq 38.6^\circ$. Therefore, in this particular basis only one mixing angle seems needed to describe the data at the current experimental accuracy. If, for the exercise, we force this equality (which is *not* based in theory), $\phi_q = \phi_s \equiv \phi$, the result of the fit is

$$f_q = (1.07 \pm 0.03) f_\pi\,, \qquad f_s = (1.37 \pm 0.27) f_\pi\,, \qquad \phi = (39.0 \pm 1.7)^\circ\,, \tag{3.9}$$

which is in agreement with earlier results [18]. The value of the mixing angle ϕ is fixed from the ratio $R_{J/\psi}$. Keeping however $f_q = f_\pi$, as predicted by the OZI-rule, leads to a poor fit with $f_s = (1.39 \pm 0.26) f_\pi$ and $\phi = (39.8 \pm 1.5)^\circ$.

Let us mention at this stage that the fact of describing in terms of one or two angles is in itself not a relevant theoretical consideration, and that the resulting physical description should of course not depend upon the choice of the initial basis. In a general formulation, there are indeed two angles. Furthermore, the choice of the starting point (octet-singlet or magic mixing) is in no way obvious. The relevant symmetry, at least for the η-η' part of the present paper, is U(3), and subject to two types of breakings, one via the strong anomaly (inducing the SU(3)-singlet term, which can also be seen as an OZI-rule violation), the other via the strange quark current mass. The effect of both breakings is similar in size in the mass matrix, leading to a mixing structure somewhere between the two basis. The J/ψ decays are of course also clear OZI violations in this framework.

4. V-P electromagnetic form factors in the two mixing angle scheme

In this Section, we want to extend our analysis to the V-P electromagnetic form factors. In particular, we are interested in the couplings of the radiative decays of lowest-lying vector mesons, $V \to (\eta, \eta')\gamma$, and of the radiative decays $\eta' \to V\gamma$, with $V = \rho, \omega, \phi$. In order to predict such couplings we follow closely the method presented in ref. [4] where the description of the light vector meson decays is based on their relation with the AVV triangle anomaly, A and V being an axial-vector and a vector current respectively. The approach both includes $\mathrm{SU}_f(3)$ breaking effects and fixes the vertex couplings $g_{VP\gamma}$ as explained below.

In that framework, one starts considering the correlation function

$$i \int d^4 x e^{iq_1 x} \langle P(q_1+q_2)| T J_\mu^{\rm EM}(x) J_\nu^V(0)|0\rangle = \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta F_{VP\gamma}\left(q_1^2, q_2^2\right) , \tag{4.1}$$

where the currents are defined as

$$\begin{aligned} J_\mu^{\rm EM} &= \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s \,, \\ J_\mu^{\rho,\omega} &= \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu u \mp \bar{d}\gamma_\mu d) \qquad \text{and} \qquad J_\mu^\phi = -\bar{s}\gamma_\mu s \,. \end{aligned} \tag{4.2}$$

The form factors values $F_{VP\gamma}(0,0)$ are fixed by the AVV triangle anomaly (one V being an electromagnetic current), and are written in terms of the pseudoscalar decay constants and the ϕ-ω mixing angle θ_V as[4]

$$\begin{aligned} F_{\rho\eta\gamma}(0,0) &= \frac{\sqrt{3}}{4\pi^2}\frac{f_{\eta'}^0 - \sqrt{2}f_{\eta'}^8}{f_{\eta'}^0 f_\eta^8 - f_{\eta'}^8 f_\eta^0} \,, \\ F_{\rho\eta'\gamma}(0,0) &= \frac{\sqrt{3}}{4\pi^2}\frac{f_\eta^0 - \sqrt{2}f_\eta^8}{f_\eta^0 f_{\eta'}^8 - f_\eta^8 f_{\eta'}^0} \,, \\ F_{\omega\eta\gamma}(0,0) &= \frac{1}{2\sqrt{2}\pi^2}\frac{(c\theta_V - s\theta_V/\sqrt{2})f_{\eta'}^0 - s\theta_V f_{\eta'}^8}{f_{\eta'}^0 f_\eta^8 - f_{\eta'}^8 f_\eta^0} \,, \\ F_{\omega\eta'\gamma}(0,0) &= \frac{1}{2\sqrt{2}\pi^2}\frac{(c\theta_V - s\theta_V/\sqrt{2})f_\eta^0 - s\theta_V f_\eta^8}{f_\eta^0 f_{\eta'}^8 - f_\eta^8 f_{\eta'}^0} \,, \\ F_{\phi\eta\gamma}(0,0) &= -\frac{1}{2\sqrt{2}\pi^2}\frac{(s\theta_V + c\theta_V/\sqrt{2})f_{\eta'}^0 + c\theta_V f_{\eta'}^8}{f_{\eta'}^0 f_\eta^8 - f_{\eta'}^8 f_\eta^0} \,, \\ F_{\phi\eta'\gamma}(0,0) &= -\frac{1}{2\sqrt{2}\pi^2}\frac{(s\theta_V + c\theta_V/\sqrt{2})f_\eta^0 + c\theta_V f_\eta^8}{f_\eta^0 f_{\eta'}^8 - f_\eta^8 f_{\eta'}^0} \,. \end{aligned} \tag{4.4}$$

Using their analytic properties, we can express these form factors by dispersion relations in the momentum of the vector current, which are then saturated with the lowest-lying resonances:

$$F_{VP\gamma}(0,0) = \frac{f_V}{m_V} g_{VP\gamma} + \cdots \,, \tag{4.5}$$

where the dots stand for higher resonances and multiparticle contributions to the correlation function. In the following we assume vector meson dominance (VMD) and thus neglect these contributions (see ref. [4] for further details).

[4]The ϕ-ω mixing angle in the octet-singlet basis is defined as

$$\begin{aligned} \phi &= \cos\theta_V \omega_8 - \sin\theta_V \omega_0 \,, \\ \omega &= \sin\theta_V \omega_8 + \cos\theta_V \omega_0 \,, \end{aligned} \tag{4.3}$$

with $\omega_8 \equiv \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$ and $\omega_0 \equiv \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$. Experimentally, the value of the vector mixing angle is measured to be above its ideal value and is $\theta_V = (38.7 \pm 0.2)^\circ$ [29]. The value obtained from a quadratic Gell-Mann–Okubo mass formula for vectors mesons is $\theta_V = (38.6 \pm 0.4)^\circ$ [1].

V	P	$g_{VP\gamma}$ (th.)		$g_{VP\gamma}$ (exp.)
ρ	η	$\frac{\sqrt{3}m_\rho}{4\pi^2 f_\rho}\frac{c\theta_0/f_8-\sqrt{2}s\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}$	$=(1.48\pm0.08)\,\mathrm{GeV}^{-1}$	$(1.59\pm0.11)\,\mathrm{GeV}^{-1}$
ρ	η'	$\frac{\sqrt{3}m_\rho}{4\pi^2 f_\rho}\frac{s\theta_0/f_8+\sqrt{2}c\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}$	$=(1.23\pm0.08)\,\mathrm{GeV}^{-1}$	$(1.35\pm0.06)\,\mathrm{GeV}^{-1}$
ω	η	$\frac{m_\omega}{2\sqrt{2}\pi^2 f_\omega}\frac{(c\theta_V-s\theta_V/\sqrt{2})c\theta_0/f_8-s\theta_V s\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}$	$=(0.57\pm0.04)\,\mathrm{GeV}^{-1}$	$(0.46\pm0.02)\,\mathrm{GeV}^{-1}$
ω	η'	$\frac{m_\omega}{2\sqrt{2}\pi^2 f_\omega}\frac{(c\theta_V-s\theta_V/\sqrt{2})s\theta_0/f_8+s\theta_V c\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}$	$=(0.56\pm0.04)\,\mathrm{GeV}^{-1}$	$(0.46\pm0.03)\,\mathrm{GeV}^{-1}$
ϕ	η	$-\frac{m_\phi}{2\sqrt{2}\pi^2 f_\phi}\frac{(s\theta_V+c\theta_V/\sqrt{2})c\theta_0/f_8+c\theta_V s\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}$	$=(-0.76\pm0.04)\,\mathrm{GeV}^{-1}$	$(-0.690\pm0.008)\,\mathrm{GeV}^{-1}$
ϕ	η'	$-\frac{m_\phi}{2\sqrt{2}\pi^2 f_\phi}\frac{(s\theta_V+c\theta_V/\sqrt{2})s\theta_0/f_8-c\theta_V c\theta_8/f_0}{c\theta_0 c\theta_8+s\theta_8 s\theta_0}$	$=(0.86\pm0.05)\,\mathrm{GeV}^{-1}$	$(0.71\pm0.04)\,\mathrm{GeV}^{-1}$

Table 1: Theoretical and experimental values of the on-shell V-(η,η') electromagnetic vertex couplings in the octet-singlet η-η' mixing angle scheme. For $g_{VP\gamma}$ (th.) we give the experimental errors coming from the decay constants $f_{P,V}$ and the mixing angle values θ_8 and θ_0. We use $\theta_V=(38.7\pm0.2)^\circ$ for the ϕ-ω mixing angle. Experimental values are taken from [1].

The f_V are the leptonic decay constants of the vector mesons, and defined by

$$\langle 0|J_\mu^V|V(p,\lambda)\rangle = m_V f_V \varepsilon_\mu^{(\lambda)}(p)\,, \tag{4.6}$$

where m_V and λ are the mass and the helicity state of the vector meson. The f_V can be determined from the experimental decay rates [1] via

$$\Gamma(V\to e^+e^-) = \frac{4\pi}{3}\alpha^2\frac{f_V^2}{m_V}c_V^2\,, \tag{4.7}$$

with $c_V=(1/\sqrt{2}, s\theta_V/\sqrt{6}, c\theta_V/\sqrt{6})$ for $V=\rho,\omega,\phi$. The experimental values are

$$\begin{aligned} f_{\rho^0} &= (221\pm2)\,\mathrm{MeV}\,,\\ f_\omega &= (180\pm3)\,\mathrm{MeV}\,,\\ f_\phi &= (239\pm4)\,\mathrm{MeV}\,. \end{aligned} \tag{4.8}$$

Finally, we introduce the vertex couplings $g_{VP\gamma}$, which are just the on-shell V-P electromagnetic form factors:

$$\langle P(p_P)|J_\mu^{\mathrm{EM}}|V(p_V,\lambda)\rangle|_{(p_V-p_P)^2=0} = -g_{VP\gamma}\epsilon_{\mu\nu\alpha\beta}p_P^\nu p_V^\alpha\varepsilon_V^\beta(\lambda)\,. \tag{4.9}$$

The decay widths of $P\to V\gamma$ and $V\to P\gamma$ are

$$\begin{aligned} \Gamma(P\to V\gamma) &= \frac{\alpha}{8}g_{VP\gamma}^2\left(\frac{m_P^2-m_V^2}{m_P}\right)^3\,,\\ \Gamma(V\to P\gamma) &= \frac{\alpha}{24}g_{VP\gamma}^2\left(\frac{m_V^2-m_P^2}{m_V}\right)^3\,. \end{aligned} \tag{4.10}$$

Eq. (4.5) allows us to identify the $g_{VP\gamma}$ couplings defined in (4.9) with the form factors $F_{VP\gamma}(0,0)$ listed in (4.4). The couplings are expressed in terms of the octet and singlet mixing angles θ_8 and θ_0, the pseudoscalar decay constants f_8 and f_0, the ϕ-ω mixing angle θ_V, and the corresponding vector decay constants f_V. These theoretical expressions are shown in table 1. We also include a numerical prediction for each coupling that should be compared with the experimental values extracted from (4.10) and ref. [1]. In the numerical analysis we have taken into account a value for the vector mixing angle of $\theta_V = (38.7 \pm 0.2)^\circ$ [29]. Our predictions are obtained from the results in eq. (3.8). The error quoted in table 1 does not reflect the full theoretical uncertainty, but namely propagates the errors from (3.8) and (4.8). The agreement between our theoretical predictions and the experimental values is quite remarkable with exceptions in the $\omega\eta\gamma$ and $\omega\eta'\gamma$ cases. However, these two couplings merit some explanation. On one case, the experimental value for $g_{\omega\eta\gamma}$ has changed from $(0.53 \pm 0.05)\,\mathrm{GeV}^{-1}$ of the PDG'02 [30] to the current $(0.46 \pm 0.02)\,\mathrm{GeV}^{-1}$ due to the exclusion of the measurement based on $e^+e^- \to \eta\gamma$ by Dolinsky et. al. [29]. On the other case, the $g_{\omega\eta'\gamma}$ coupling is rather sensitive to the ϕ-ω mixing angle; for instance setting θ_V to the ideal mixing value of 35.3° reduces the coupling by a 10%. As seen from table 1, the predictions for the decays $\phi \to (\eta, \eta')\gamma$, which are the best measured and are largely independent of θ_V, are in good agreement with data. It is worth noting that the comparison shown in table 1 is performed with $f_8 = 1.34 f_\pi$. Using instead $f_8 = 1.28 f_\pi$ one obtains similar numbers with the exception of $g_{\phi\eta\gamma} = (-0.80 \pm 0.04)\,\mathrm{GeV}^{-1}$ and $g_{\phi\eta'\gamma} = (0.91 \pm 0.06)\,\mathrm{GeV}^{-1}$ which are worse fitted. In this sense, the experimental data seem to prefer a value for f_8 higher than the one predicted by standard χPT. Once again, the 2-angle calculation is the relevant one. For completeness however, we have included in the table below same comparison but fixing $\theta_8 = \theta_0 \equiv \theta$. In this case, the results are very similar for $f_8 = 1.28 f_\pi$ and $f_8 = 1.34 f_\pi$ but in both cases the χ^2/d.o.f. is increased by a factor of 3 as compared to the value in the two mixing angle scheme. In particular, $g_{\phi\eta'\gamma}$ are fitted to $(1.21 \pm 0.07)\,\mathrm{GeV}^{-1}$ and $(1.20 \pm 0.06)\,\mathrm{GeV}^{-1}$ respectively, which are in clear contradiction with data.

Table 1 constitutes one of the main results of our work. Our analysis shows that the assumption of saturating the form factors $F_{VP\gamma}$ by lowest-lying resonances is satisfactory (a conclusion already reached in ref. [4]), and that the η-η' system described in the two mixing angle scheme (octet-singlet basis) fits the data much better than the one mixing angle scheme does. To quantify this improvement, we have performed various fits to the full set of experimental data assuming, or not, the two mixing angle scheme of the η-η' system. The results are presented in table 2. To check the consistency of our approach we have extended the fit in eq. (3.7) or eq. (3.8) to include all experimental data that account not only for the decay widths $(\eta, \eta') \to \gamma\gamma$ and the ratio $R_{J/\psi}$ but also the radiative decay widths of $V \to P\gamma$ and $P \to V\gamma$. The theoretical constraint $f_8 = 1.28 f_\pi$ or $1.34 f_\pi$ is relaxed in order to test the dependence of the result on the value of this parameter. The experimental constrain $\theta_V = (38.7 \pm 0.2)^\circ$ is also relaxed to test the stability of the fit.

As seen from table 2, a significant improvement in the χ^2/d.o.f. is achieved when the constrain $\theta_8 = \theta_0 \equiv \theta$ is relaxed (in the most favorable case the χ^2/d.o.f. is reduced by more than a factor of 3), allowing us to show explicitly the improvement of our analysis using the two mixing angle scheme with respect to the one using the one mixing angle scheme. In general, the fits with $1.34 f_\pi$ are slightly better than those with $f_8 = 1.28 f_\pi$. To leave f_8 as a free parameter does not make any substantial difference in the one angle case while in the case of two mixing angles the value increases up to $f_8 = (1.51 \pm 0.05) f_\pi$, in disagreement with the expectations of χPT and large N_c χPT. However, the analysis

Assumptions	Results	χ^2/d.o.f.	Assumptions	Results	χ^2/d.o.f.
θ_8 and θ_0 free $f_8 = 1.28 f_\pi$ $\theta_V = (38.7 \pm 0.2)^\circ$	$\theta_8 = (-22.5 \pm 1.3)^\circ$ $\theta_0 = (-8.0 \pm 1.4)^\circ$ $f_0 = (1.21 \pm 0.03) f_\pi$	42.3/6	$\theta_8 = \theta_0 \equiv \theta$ $f_8 = 1.28 f_\pi$ $\theta_V = (38.7 \pm 0.2)^\circ$	$\theta = (-16.9 \pm 1.2)^\circ$ $f_0 = (1.15 \pm 0.03) f_\pi$	81.4/7
θ_8 and θ_0 free $f_8 = 1.34 f_\pi$ $\theta_V = (38.7 \pm 0.2)^\circ$	$\theta_8 = (-22.9 \pm 1.3)^\circ$ $\theta_0 = (-6.6 \pm 1.4)^\circ$ $f_0 = (1.23 \pm 0.03) f_\pi$	31.2/6	$\theta_8 = \theta_0 \equiv \theta$ $f_8 = 1.34 f_\pi$ $\theta_V = (38.7 \pm 0.2)^\circ$	$\theta = (-16.7 \pm 1.1)^\circ$ $f_0 = (1.16 \pm 0.03) f_\pi$	77.8/7
θ_8 and θ_0 free f_8 free $\theta_V = (38.7 \pm 0.2)^\circ$	$\theta_8 = (-23.8 \pm 1.4)^\circ$ $\theta_0 = (-2.4 \pm 1.9)^\circ$ $f_8 = (1.51 \pm 0.05) f_\pi$ $f_0 = (1.29 \pm 0.04) f_\pi$	18.8/5	$\theta_8 = \theta_0 \equiv \theta$ f_8 free $\theta_V = (38.7 \pm 0.2)^\circ$	$\theta = (-16.5 \pm 1.2)^\circ$ $f_8 = (1.37 \pm 0.05) f_\pi$ $f_0 = (1.17 \pm 0.04) f_\pi$	77.4/6
θ_8 and θ_0 free f_8 free θ_V free	$\theta_8 = (-24.0 \pm 1.6)^\circ$ $\theta_0 = (-2.5 \pm 1.9)^\circ$ $f_8 = (1.51 \pm 0.05) f_\pi$ $f_0 = (1.29 \pm 0.04) f_\pi$ $\theta_V = (39.4 \pm 2.2)^\circ$	18.6/4	$\theta_8 = \theta_0 \equiv \theta$ f_8 free θ_V free	$\theta = (-15.7 \pm 1.4)^\circ$ $f_8 = (1.37 \pm 0.05) f_\pi$ $f_0 = (1.17 \pm 0.03) f_\pi$ $\theta_V = (36.5 \pm 1.8)^\circ$	76.1/5

Table 2: Results for the η-η' mixing angles and decay constants in the octet-singlet basis of the two mixing angle scheme *(left)* and in the one mixing angle scheme *(right)*. For every fit, the theoretical assumptions taken, the set of numerical results, and the value of the χ^2/d.o.f. are shown in the first, second and third column respectively. The fitted experimental data includes the decay widths of $(\eta, \eta') \to \gamma\gamma$, $V \to P\gamma$, $P \to V\gamma$, and the ratio $R_{J/\psi}$.

of the parameter correlation coefficients reveals a strong positive correlation between f_8 and θ_0 ($+0.674$) — the latter quantity being taken as a negative number in the present convention; in other terms, the correlation to the *absolute* value of θ_0 is negative. The correlations between f_8 and θ_8 or θ_8 and θ_0 are negative and much weaker (-0.126 and -0.124, respectively). It is also observed that when f_8 is fixed, the remaining correlation coefficients are smaller. This increase is translated into a considerably better fit. Finally, it is seen in table 2 that relaxing the experimental constrain $\theta_V = (38.7 \pm 0.2)^\circ$ does not produce any effect on the fits.

Up to now, we have shown in the octet-singlet basis the need for a two mixing angle scheme in order to describe experimental data in a better way. In the following, we proceed to perform the same kind of analysis but in the quark-flavour basis. We will see that at the *current experimental* accuracy, the two angles have compatible values in this case. There is however no strong reason to impose this equality as a constraint. See also above for a discussion of the respective effects of the strange quark mass and anomalies in the mixing.

In table 3 we present the results of various fits taking into account all experimental data available — namely $(\eta, \eta') \to \gamma\gamma$, $V \to P\gamma$, $P \to V\gamma$, and the ratio $R_{J/\psi}$ — and the corresponding theoretical expressions found in eqs. (A.2)–(A.5) of the Appendix. The theoretical constrain $f_q = f_\pi$ is relaxed to test the dependence of the result on the value of

Assumptions	Results	χ^2/d.o.f.	Assumptions	Results	χ^2/d.o.f.
ϕ_q and ϕ_s free $f_q = f_\pi$ $\phi_V = (3.4 \pm 0.2)^\circ$	$\phi_q = (40.4 \pm 1.2)^\circ$ $\phi_s = (41.3 \pm 1.3)^\circ$ $f_s = (1.66 \pm 0.06) f_\pi$	34.6/6	$\phi_q = \phi_s \equiv \phi$ $f_q = f_\pi$ $\phi_V = (3.4 \pm 0.2)^\circ$	$\phi = (40.8 \pm 0.9)^\circ$ $f_s = (1.66 \pm 0.06) f_\pi$	34.9/7
ϕ_q and ϕ_s free f_q free $\phi_V = (3.4 \pm 0.2)^\circ$	$\phi_q = (39.9 \pm 1.3)^\circ$ $\phi_s = (41.4 \pm 1.4)^\circ$ $f_q = (1.09 \pm 0.03) f_\pi$ $f_s = (1.66 \pm 0.06) f_\pi$	18.8/5	$\phi_q = \phi_s \equiv \phi$ f_q free $\phi_V = (3.4 \pm 0.2)^\circ$	$\phi = (40.6 \pm 0.9)^\circ$ $f_q = (1.10 \pm 0.03) f_\pi$ $f_s = (1.66 \pm 0.06) f_\pi$	19.4/6
ϕ_q and ϕ_s free f_q free ϕ_V free	$\phi_q = (39.8 \pm 1.3)^\circ$ $\phi_s = (41.2 \pm 1.5)^\circ$ $f_q = (1.09 \pm 0.03) f_\pi$ $f_s = (1.67 \pm 0.07) f_\pi$ $\phi_V = (4.2 \pm 2.1)^\circ$	18.6/4	$\phi_q = \phi_s \equiv \phi$ f_q free ϕ_V free	$\phi = (40.4 \pm 1.0)^\circ$ $f_q = (1.10 \pm 0.03) f_\pi$ $f_s = (1.66 \pm 0.07) f_\pi$ $\phi_V = (4.1 \pm 2.2)^\circ$	19.3/5

Table 3: Results for the η-η' mixing angles and decay constants in the quark-flavour basis of the two mixing angle scheme *(left)* and in the one mixing angle scheme *(right)*. The conventions are the same as in table 2.

this parameter and the same happens with the experimental constrain $\phi_V = (3.4 \pm 0.2)^\circ$. As seen from table 3, there is no significant difference at the present experimental accuracy between the χ^2/d.o.f. of the fits when data are described in terms of two mixing angles (in the quark-flavour basis) or if $\phi_q = \phi_s \equiv \phi$ is imposed. It is however important to notice that the fit considerably improves when the parameter f_q is left free. In this case the value obtained $f_q = (1.10 \pm 0.03) f_\pi$ is incompatible with the value in the large N_c limit $f_q = f_\pi$.

5. Discussion about the mixing parameters

In this Section, we compare our best results for the pseudoscalar decay constants and mixing angles in the octet-singlet and quark-flavour basis with the theoretical expectations of large N_c χPT and previous phenomenological analyses. The value of the mixing parameters extracted from our best fit in the two mixing angle scheme are

$$\begin{aligned} f_8 &= (1.51 \pm 0.05) f_\pi\,, \qquad \theta_8 = (-23.8 \pm 1.4)^\circ\,, \\ f_0 &= (1.29 \pm 0.04) f_\pi\,, \qquad \theta_0 = (-2.4 \pm 1.9)^\circ\,, \end{aligned} \tag{5.1}$$

in the octet-singlet basis, and

$$\begin{aligned} f_q &= (1.09 \pm 0.03) f_\pi\,, \qquad \phi_q = (39.9 \pm 1.3)^\circ\,, \\ f_s &= (1.66 \pm 0.06) f_\pi\,, \qquad \phi_s = (41.4 \pm 1.4)^\circ\,, \end{aligned} \tag{5.2}$$

in the quark-flavour basis. These values are extracted from a comparison with experimental data only assuming that the pseudoscalar decay constants involved in the corresponding

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processes follow the parametrization given in eqs. (2.4) and (2.6), respectively. At the present accuracy, our results satisfy the approximate relations existing between the two different sets of mixing parameters[5] [18]:

$$f_8 = \sqrt{\frac{1}{3}f_q^2 + \frac{2}{3}f_s^2}\,, \qquad \theta_8 = \phi - \arctan\left(\frac{\sqrt{2}f_s}{f_q}\right),$$
$$f_0 = \sqrt{\frac{2}{3}f_q^2 + \frac{1}{3}f_s^2}\,, \qquad \theta_0 = \phi - \arctan\left(\frac{\sqrt{2}f_q}{f_s}\right). \tag{5.3}$$

Large N_c χPT predicts [12, 13]:

$$f_8^2 = \frac{4f_K^2 - f_\pi^2}{3}\,, \qquad f_0^2 = \frac{2f_K^2 + f_\pi^2}{3} + f_\pi^2\Lambda_1\,,$$
$$f_8 f_0 \sin(\theta_8 - \theta_0) = -\frac{2\sqrt{2}}{3}(f_K^2 - f_\pi^2)\,, \tag{5.4}$$

in the octet-singlet basis and

$$f_q^2 = f_\pi^2 + \frac{2}{3}f_\pi^2\Lambda_1\,, \qquad f_s^2 = 2f_K^2 - f_\pi^2 + \frac{1}{3}f_\pi^2\Lambda_1\,,$$
$$f_q f_s \sin(\phi_q - \phi_s) = \frac{\sqrt{2}}{3}f_\pi^2\Lambda_1\,, \tag{5.5}$$

in the quark-flavour basis [26]. These expressions are valid at next-to-leading order in the large N_c χPT expansion where the octet-singlet (and quark-flavour) pseudoscalar decay constants can be written in terms of the known f_π and f_K decay constants and the unknown OZI-rule violating parameter Λ_1. Using the experimental constrain $f_K = 1.22f_\pi$, one obtains[6] [12]: $f_8 = 1.28f_\pi$, $\theta_8 = -20.5^\circ$, $f_0 \simeq 1.25$, and $\theta_0 \simeq -4^\circ$. Our results in eq. (5.1) are quite in agreement with the former values except for the case of f_8. Note however that in ref. [12] the value of f_8 is fixed from theory while in our analysis it is fitted from a direct comparison with experimental data where a positive correlation between f_8 and θ_0 appears. For $f_8 = 1.34f_\pi$ the results of the fit are in perfect agreement with the predictions from large N_c χPT even though the quality of the fit is slightly reduced.

As seen from table 2, if the constrain $f_8 = 1.28f_\pi$ is imposed one gets a worse fit. The same kind of comparison can be performed in the quark-flavour basis. From the phenomenological analysis [18] the values $f_q = (1.07 \pm 0.02)f_\pi$, $f_s = (1.34 \pm 0.06)f_\pi$ and $\phi = (39.3 \pm 1.0)^\circ$ are obtained. In this case, our results in eq. (5.2) fairly agree with the exception of f_s which clearly disagrees. This difference may be due to the fact that the analysis in ref. [18] is based on a different set of experimental data not including for instance the very precise and recent $\phi \to (\eta, \eta')\gamma$ decays which are very dependent on f_s (see eq. (A.5)).

[5] The relations (5.3) are obtained once the non-strange and strange axial-vector currents are written in terms of the octet and singlet ones (see eq. (2.7)) and are valid for $\phi_q = \phi_s \equiv \phi$.

[6] A value of $f_8 = 1.34f_\pi$ is obtained if chiral logs and higher order contributions are also taken into account [13].

Our best results for the mixing parameters in eqs. (5.1) and (5.2) can be used to check the consistency of eqs. (5.4) and (5.5), and therefore to test the reliability on the large N_c χPT framework. Accordingly, our fitted values for f_8 and f_0 together with the third equation in (5.4) can be used to get $\theta_8-\theta_0=(-13.7\pm0.6)^\circ$ as a prediction for the difference of the two mixing angles in the octet-singlet basis. If one compares this prediction with our result $\theta_8-\theta_0=(-21.4\pm2.4)^\circ$ a disagreement is again obtained. Using $f_8=1.34f_\pi$ and our fitted value for f_0 one gets $\theta_8-\theta_0=(-16.2\pm0.4)^\circ$ to compare with our result $\theta_8-\theta_0=(-16.3\pm1.9)^\circ$ (see table 2). The second equation in (5.4) may be used to get a prediction for the OZI-rule violating parameter Λ_1 once f_0 is provided. Using our value for f_0 one gets $\Lambda_1=0.34\pm0.10$. Λ_1 can also be extracted from the set of equations (5.5). From the fitted values for $f_{q,s}$ and $\phi_{q,s}$ the results $\Lambda_1=0.32\pm0.10$, 2.34 ± 0.60 and -0.10 ± 0.13 are obtained using the first, second and third equation in (5.5), respectively. Indeed, the same behaviour is also observed in the phenomenological analysis of ref. [18]. The values obtained for the mixing parameters in this analysis lead to values for Λ_1 that are incompatible with the constrain $\Lambda_1\equiv0$ imposed in their fit.

The numbers obtained in the former discussion hint at a disagreement with large N_c χPT. Some care should be taken however to qualify this statement. On the one hand, the statistical significance is still limited, but will obviously improve as critical channels are measured more accurately. On the other hand, it is plainly clear that the critical information comes here from radiative decays of vector mesons, where we use vector meson dominance. Questions have been raised as to the consistency of vector resonances and VMD with Chiral Perturbation Theory and short distance QCD cf. e.g. [31]. This latter remark should however be moderated by the fact that our approach was carefully tested in the charged mesons sector (away from the eta mixing problems) [4].

6. Conclusions

In this work we have performed a phenomenological analysis on various decay processes using a two mixing angle scheme for the η-η' system. First we have used the radiative decays $(\eta,\eta')\to\gamma\gamma$ together with the ratio $R_{J/\psi}$ to fit the values of the pseudoscalar decay constants and the mixing angles in both the octet-singlet and the quark-flavour basis. Using the description of vector meson decays in terms of their relation with the AVV triangle anomaly, a theoretical prediction for the $g_{VP\gamma}$ couplings have been derived. The agreement between our theoretical predictions and the experimental values is very good and can be considered as a consistency check of the whole approach. Second we have extend our analysis to include the $V\to P\gamma$ and $P\to V\gamma$ decays in the fits showing that a two mixing angle description in the octet-singlet basis is definitely required in order to achieve good agreement with experimental data. On the contrary, in the quark-flavour basis and with the present experimental accuracy a one mixing angle description of the processes is still enough to reach agreement. Finally, we have compared our best fitted mixing parameters with the predictions from large N_c χPT showing a possible discrepancy with this framework. Higher accuracy data and more refined theoretical analyses would contribute to clarify the preceding issue.

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A. Theoretical expressions in the quark-flavour basis

In the quark-flavour basis, the interpolating fields η and η' are expressed as

$$\eta(x) = \frac{1}{m_\eta^2}\frac{f_{\eta'}^s\partial^\mu A_\mu^q(x) - f_{\eta'}^q\partial^\mu A_\mu^s(x)}{f_{\eta'}^s f_\eta^q - f_{\eta'}^q f_\eta^s}\,, \qquad \eta'(x) = \frac{1}{m_{\eta'}^2}\frac{f_\eta^s\partial^\mu A_\mu^q(x) - f_\eta^q\partial^\mu A_\mu^s(x)}{f_\eta^s f_{\eta'}^q - f_\eta^q f_{\eta'}^s}\,, \tag{A.1}$$

the $(\eta,\eta')\to\gamma\gamma$ decay widths as

$$\begin{aligned}
\Gamma(\eta\to\gamma\gamma) &= \frac{\alpha^2 m_\eta^3}{32\pi^3}\left(\frac{C_q f_{\eta'}^s - C_s f_{\eta'}^q}{f_{\eta'}^s f_\eta^q - f_{\eta'}^q f_\eta^s}\right)^2 = \frac{\alpha^2 m_\eta^3}{32\pi^3}\left(\frac{C_q c\phi_s/f_q - C_s s\phi_q/f_s}{c\phi_s c\phi_q + s\phi_q s\phi_s}\right)^2\,,\\
\Gamma(\eta'\to\gamma\gamma) &= \frac{\alpha^2 m_{\eta'}^3}{32\pi^3}\left(\frac{C_q f_\eta^s - C_s f_\eta^q}{f_\eta^s f_{\eta'}^q - f_\eta^q f_{\eta'}^s}\right)^2 = \frac{\alpha^2 m_{\eta'}^3}{32\pi^3}\left(\frac{C_q s\phi_s/f_q + C_s c\phi_q/f_s}{c\phi_s c\phi_q + s\phi_q s\phi_s}\right)^2\,,
\end{aligned} \tag{A.2}$$

with $C_q = 5/3$ and $C_s = \sqrt{2}/3$, and the ratio $R_{J/\psi}$ as

$$R_{J/\psi} = \left|\frac{m_{\eta'}^2 f_{\eta'}^q}{m_\eta^2 f_\eta^q}\right|^2\left(\frac{p_{\eta'}}{p_\eta}\right)^3 = \tan^2\phi_q\left(\frac{m_{\eta'}}{m_\eta}\right)^4\left(\frac{p_{\eta'}}{p_\eta}\right)^3\,. \tag{A.3}$$

The form factors $F_{VP\gamma}(0,0)$ are written in terms of the pseudoscalar decay constants (f_q, f_s) and the ϕ-ω mixing angle in the quark-flavour basis ($\phi_V = \theta_V - \arctan\frac{1}{\sqrt{2}}$):

$$\begin{aligned}
F_{\rho\eta\gamma}(0,0) &= \frac{3}{4\pi^2}\frac{f_{\eta'}^s}{f_{\eta'}^s f_\eta^q - f_{\eta'}^q f_\eta^s}\,, & F_{\rho\eta'\gamma}(0,0) &= \frac{3}{4\pi^2}\frac{f_\eta^s}{f_\eta^s f_{\eta'}^q - f_\eta^q f_{\eta'}^s}\,,\\
F_{\omega\eta\gamma}(0,0) &= \frac{1}{4\pi^2}\frac{c\phi_V f_{\eta'}^s - 2s\phi_V f_{\eta'}^q}{f_{\eta'}^s f_\eta^q - f_{\eta'}^q f_\eta^s}\,, & F_{\omega\eta'\gamma}(0,0) &= \frac{1}{4\pi^2}\frac{c\phi_V f_\eta^s - 2s\phi_V f_\eta^q}{f_\eta^s f_{\eta'}^q - f_\eta^q f_{\eta'}^s}\,,\\
F_{\phi\eta\gamma}(0,0) &= -\frac{1}{4\pi^2}\frac{s\phi_V f_{\eta'}^s + 2c\phi_V f_{\eta'}^q}{f_{\eta'}^s f_\eta^q - f_{\eta'}^q f_\eta^s}\,, & F_{\phi\eta'\gamma}(0,0) &= -\frac{1}{4\pi^2}\frac{s\phi_V f_\eta^s + 2c\phi_V f_\eta^q}{f_\eta^s f_{\eta'}^q - f_\eta^q f_{\eta'}^s}\,.
\end{aligned} \tag{A.4}$$

Finally, the vertex couplings $g_{VP\gamma}$ in this basis are

$$\begin{aligned}
g_{\rho\eta\gamma} &= \frac{3m_\rho}{4\pi^2 f_\rho}\frac{c\phi_s/f_q}{c\phi_s c\phi_q + s\phi_q s\phi_s} \simeq \frac{3m_\rho}{4\pi^2 f_\rho}\frac{c\phi}{f_q}\,,\\
g_{\rho\eta'\gamma} &= \frac{3m_\rho}{4\pi^2 f_\rho}\frac{s\phi_s/f_q}{c\phi_s c\phi_q + s\phi_q s\phi_s} \simeq \frac{3m_\rho}{4\pi^2 f_\rho}\frac{s\phi}{f_q}\,,
\end{aligned}$$

$$
\begin{aligned}
g_{\omega\eta\gamma} &= \frac{m_\omega}{4\pi^2 f_\omega}\frac{c\phi_V c\phi_s/f_q - 2s\phi_V s\phi_q/f_s}{c\phi_s c\phi_q + s\phi_q s\phi_s} \simeq \frac{m_\omega}{4\pi^2 f_\omega}\left(c\phi_V\frac{c\phi}{f_q} - 2s\phi_V\frac{s\phi}{f_s}\right), \\
g_{\omega\eta'\gamma} &= \frac{m_\omega}{4\pi^2 f_\omega}\frac{c\phi_V s\phi_s/f_q + 2s\phi_V c\phi_q/f_s}{c\phi_s c\phi_q + s\phi_q s\phi_s} \simeq \frac{m_\omega}{4\pi^2 f_\omega}\left(c\phi_V\frac{s\phi}{f_q} + 2s\phi_V\frac{c\phi}{f_s}\right), \\
g_{\phi\eta\gamma} &= -\frac{m_\phi}{4\pi^2 f_\phi}\frac{s\phi_V c\phi_s/f_q + 2c\phi_V s\phi_q/f_s}{c\phi_s c\phi_q + s\phi_q s\phi_s} \simeq -\frac{m_\phi}{4\pi^2 f_\phi}\left(s\phi_V\frac{c\phi}{f_q} + 2c\phi_V\frac{s\phi}{f_s}\right), \\
g_{\phi\eta'\gamma} &= -\frac{m_\phi}{4\pi^2 f_\phi}\frac{s\phi_V s\phi_s/f_q - 2c\phi_V c\phi_q/f_s}{c\phi_s c\phi_q + s\phi_q s\phi_s} \simeq -\frac{m_\phi}{4\pi^2 f_\phi}\left(s\phi_V\frac{s\phi}{f_q} - 2c\phi_V\frac{c\phi}{f_s}\right), \quad \text{(A.5)}
\end{aligned}
$$

where the approximations are valid for $\phi_q = \phi_s \equiv \phi$.

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